K-coverage Formation for Target Tracking Using Nonholonomic Agents

Qiang Shao, Jiangping Hu*
School of Automation Engineering
University of Electronic Science and Technology of China
Chengdu, China
hjp_lzu@163.com

Abstract—Formation control is an important problem in multi-agent systems. In this paper, it is assumed that a target is moving with a disturbed dynamics, and then, a k-coverage formation control is designed for every nonholonomic agent to improve tracking qualities. A virtual agent approach is adopted in the formation control design. The stability of the tracking error system is analyzed. Finally, some numerical simulations are given to validate the proposed tracking algorithms.

Keywords—formation control, k-coverage, nonholonomic dynamics, multi-agent systems, target tracking

I. INTRODUCTION

In recent years, formation control problems of multi-agent systems have been studied intensely due to its wide applications in mobile robotics [1]–[3], air traffic control [4], [5], satellite clustering [6], automatic highways [7], and mobile sensor networks [8], [9]. The existing approaches to formation control generally fall into two categories. The one is the leader-follower approach [10], [11] and the other is the virtual leader approach [12], [13]. The combination of these two categories of approaches is adopted in this paper to deal with a target tracking problem with nonholonomic agents.

In many real applications, the leader or target is moving with a disturbed dynamics. When the target dynamics is completely unknown, random walk model can be used. Thus, it is crucial to design a feasible target tracking algorithm. It is well known that some tracking filters, for example, the extended Kalman filter (EKF), unscented Kalman filter (UKF), particle filter (PF), have been developed for nonlinear target dynamics. However, most filters suffer from convergence guarantees [14] and tracking qualities [15]. Especially, the integration of tracking filter (such as EKF) and formation control improves the tracking performance effectively [15].

Inspired by the observation, we focus on giving a thorough investigation of formation control in target tracking in this paper. The trajectory of the target is assumed partially unknown, which is described by an exogenous system. The dynamics of all mobile agents are second-order nonholonomic systems. A requirement on the formation controls of all agents is to ensure that the target is monitored by at least k agents at any time, which is called a k-coverage problem [16]. When the target and agents are restricted in a second-dimensional plane, a neighbor-based formation controller is proposed for each agent to achieve an equilateral-triangle deployment and, thus, solve the 3-coverage formation control problem.

The rest of the paper is organized as follows. In Section 2, a target tracking problem is formulated with some graph preliminaries. In Section 3, k-coverage formation controllers are designed to solve the 3-corverage target tracking problem. The stability of the tracking error system is analyzed in Section 4. Finally, some numerical results are given to validate the proposed tracking algorithms in Section 5.

II. PROBLEM FORMATION

In this section, a target tracking problem is formulated for a group of autonomous mobile agents.

Consider N agents and a target with partially unknown dynamics in a second dimensional tracking region. Each agent is occupied by a disk $\Omega_i = \{s \in R^2 \mid || s|| - s_i \le r\}, i = 1, \cdots, N$, where $s_i = col(x_i, y_i) \in R^2$ is the position vector and r is the sensing range of agent i. The interconnection network of the agents is described by a directed tree G = (V, E), where the node set $V = \{1, \dots, N\}$ and the arc set $E = V \times V$, which is shown as in Fig. 1. The node i represents agent i. The arc $(i, i-1) \in E$, if and only if agent i can receive information from agent i-1. For convenience, let the target be labeled by 0.

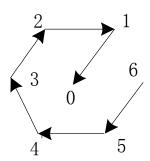


Fig.1 Tree interconnection topology

The kinematics of each agent is described by a nonholonomic nonlinear system with pure rolling and nonslipping:

$$\begin{cases} \dot{x}_{s,i} = v_{s,i} \cos \varphi_i, \\ \dot{y}_{s,i} = v_{s,i} \sin \varphi_i, \quad i = 1, ..., N \\ \dot{\varphi}_i = w_i, \end{cases}$$
 (1)

where $col(x_i(t), y_i(t)) \in R^2$, $v_{s,i}(t)$, $\varphi_i(t)$ and $w_i(t)$ are the position state, translational velocity, angular and angular velocity of agent i, respectively. Let $z_i = col(x_i, y_i, \varphi_i)$ be the triple coordinate of agent i. Here, the controls to be designed are $v_{s,i}(t)$ and $w_i(t)$

Suppose the dynamics of the target is described by a second order linear system:

$$\begin{cases} \dot{p}(t) = q(t), \\ \dot{q}(t) = u(t), \end{cases}$$
 (2)

 $\begin{cases} \dot{p}(t)=q(t),\\ \dot{q}(t)=u(t), \end{cases}$ where $p\in R^2, q\in R^2$ are, respectively, the position, velocity of the target, $u(t) \in \mathbb{R}^2$ is a partially unknown disturbance. The disturbance is modelled by a linear exogenous system

$$\begin{cases} \dot{\varsigma}(t) = \Gamma \varsigma(t), \\ u(t) = D \varsigma(t), \end{cases}$$
 (3)

where Γ , F, D are known appropriate dimensional matrices. If Γ , D have negative real-part eigenvalues, the disturbance will approach zero as time goes to infinity. Without loss of generality, it is assumed that the eigenvalues of Γ have nonnegative real-parts.

In order to track the target with high quality, k-coverage methodology is realized by designing formation controls $v_{s,i}(t)$ and $w_i(t)$ for i=1,...,N. Following the literature on formation control [17], the desired formation can be encoded in terms of a formation graph.

Definition 1 (Formation graph): The formation graph $G_F = (V, E_F, D)$ is an undirected graph that consists of (i) a set of vertices $V = \{1, ..., N\}$; (ii) a set of edges, $E_F = V \times V$ containing pairs of nodes that represent inter-agent formation specifications and (iii) a set of desired configurations $D = \{d_{ij}\}$ that specify the desired inter-agent relative position.

Further, a k-coverage formation requires all agents to achieve a desired formation graph and the target is sensed by at least k agents, simultaneously. Thus, we say that a k-coverage formation problem is solved if $\|z\| - z_j - d_{ij} \to 0$ and a bounded k-coverage formation problem is solved if $\|z_j\| - z_j - d_{ij} < \varepsilon$ for some positive constant ε . The objective of each agent is to be stabilized to a desired distance and angular with respect to its neighboring agents, which solves the k-coverage formation problem during the target tracking process.

CONTROL DESIGN

In this section, formation controllers $v_{s,i}(t)$ and $w_i(t)$ in (1) for i=1... N, are designed based on the path-following control algorithm presented in [12].

A reference point $(x_{s,i}^0, y_{s,i}^0)$ is chosen as the virtual agent

for agent i at a pre-assigned distance d_{0i} and bearing angular eta_{0i} Let d_i denote the actual distance between agent i and agent i-1 and β_i denote the actual bearing angle from the moving direction to the adjacent edge $S_i S_{i-1}$ (see Fig.2). The bearing satisfy $\beta_i, \beta_{0,i} \in [-\pi, -\pi/2) \cup$ angles $\cup (\pi/2,\pi]$. Then, the bounded k-coverage formation problem is solved if $\|d_i - d_{0i}\| < \varepsilon_1$, $\|\beta_i - \beta_{0i}\| < \varepsilon_2$, for any positive numbers \mathcal{E}_1 and \mathcal{E}_2 , as time goes to infinity.

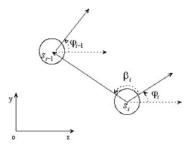


Fig.2 Notations of angles and distances

Before we design the formation control $V_{s,i}$ and W_i for agent i, a transformation is exerted to system (1) firstly.

Since

$$d_i^2 = (x_{s,i} - x_{s,i-1})^2 + (y_{s,i} - y_{s,i-1})^2$$

one has

$$\dot{d}_i = -v_{s,i} \cos \beta_i + v_{s,i-1} \cos(\theta_i + \beta_i) \tag{4}$$

From Fig.2, it can be seen that

$$\beta_{i} = \pi - \varphi_{i} - \gamma_{i},$$

$$\gamma_{i} = \arctan(\frac{(y_{s,i-1} - y_{s,i})}{(x_{s,i} - x_{s,i-1})})$$
where

Then,

$$\dot{\gamma}_i = \frac{1}{d_i} v_{s,i-1} \sin(\varphi_i + \beta_i) - \frac{1}{d_i} v_{s,i} \sin \beta_i$$
 (5)

 $\dot{\beta}_i = -\dot{\varphi}_i - \dot{\gamma}_i$ $=-w_i-\frac{1}{d_i}v_{s,i-1}\sin(\varphi_i+\beta_i)+\frac{1}{d_i}v_{s,i}\sin\beta_i$

Define $\theta_i = \varphi_i - \varphi_{i-1}$, one has

$$\dot{\theta}_i = w_i - w_{i-1},\tag{7}$$

Thus, from (4), (5) and (6), system (1) is transformed to a new version:

$$\begin{cases} \dot{d}_{i} = -v_{s,i}\cos\beta_{i} + v_{s,i-1}\cos(\theta_{i} + \beta_{i}) \\ \dot{\theta}_{i} = w_{i} - w_{i-1} \\ \dot{\beta}_{i} = -w_{i} - \frac{1}{d_{i}}v_{s,i-1}\sin(\theta_{i} + \beta_{i}) + \frac{1}{d_{i}}v_{s,i}\sin\beta_{i} \end{cases}$$
(8)

(6)

for i=1,...,N.

Next, the k-coverage formation tracking controllers $v_{s,i}$ and w_i are to be designed for transformed system (8).

Regarding the following fact,

$$\begin{cases} x_{s,i}^{0} = x_{s,i} + d_{0,i} \cos(\varphi_{i} + \beta_{0,i}) \\ y_{s,i}^{0} = y_{s,i} + d_{0,i} \sin(\varphi_{i} + \beta_{0,i}) \end{cases}$$
(9)

one has

$$\begin{cases} v_{s,i} = \frac{\dot{x}_{s,i}^{0} \cos(\varphi_{i} + \beta_{0i}) + \dot{y}_{s,i}^{0} \sin(\varphi_{i} + \beta_{0i})}{\cos \beta_{0i}} \\ w_{i} = \frac{-\dot{x}_{s,i}^{0} \sin \varphi_{i} + \dot{y}_{s,i}^{0} \cos \varphi_{i}}{d_{0i} \cos \beta_{0i}} - \dot{\beta}_{0i} \end{cases}$$

or,

$$\begin{pmatrix} v_{s,i} \\ w_i \end{pmatrix} = \begin{pmatrix} \cos(\varphi_i + \beta_{0i}) & \sin(\varphi_i + \beta_{0i}) \\ \cos\beta_{0i} & \cos\beta_{0i} \\ -\frac{\sin\varphi_i}{d_{0i}\cos\beta_{0i}} & \frac{\cos\varphi_i}{d_{0i}\cos\beta_{0i}} \end{pmatrix} \begin{pmatrix} \dot{x}_{s,i}^0 \\ \dot{y}_{s,i}^0 \end{pmatrix} - \begin{pmatrix} 0 \\ \dot{\beta}_{0i} \end{pmatrix}$$
(10)

Remark 1: When the target changes its direction slowly, β_{0i} can be chosen as constants. However, once the target changes its direction very quickly, it is not necessary to require all agents to change their directions so quickly as to keep the desired formation. In order to reduce the response time of the whole group, the formation of agents can change the direction in a smooth way according to the following dynamics designed for

the desired bearing angle $\beta_{0,i}$:

$$\dot{\beta}_{0,i} = \begin{cases}
0 & if \left| \frac{\dot{x}_0 \ddot{y}_0 - \dot{y}_0 \ddot{x}_0}{\dot{x}_0^2 + \dot{y}_0^2} \right| \leq \mu, \\
-\frac{\rho}{1 + f(t)} & if \frac{\dot{x}_0 \ddot{y}_0 - \dot{y}_0 \ddot{x}_0}{\dot{x}_0^2 + \dot{y}_0^2} > \mu, \\
\frac{\rho}{1 + f(t)} & otherewise.
\end{cases} (11)$$

where (x_0,y_0) is the position of the target, f(t) is an appropriate monotone increasing function with respect to t, the threshold $\mu>0$ and the scaling factor $0\leq\rho\leq1$.

Furthermore, if $\dot{x}_{s,i}^0$, $\dot{y}_{s,i}^0$ are choose as

$$\dot{x}_{s,i}^{0} = -k_{i}(x_{s,i}^{0} - x_{s,i-1}) + \dot{x}_{s,i-1},
\dot{y}_{s,i}^{0} = -k_{i}(y_{s,i}^{0} - y_{s,i-1}) + \dot{y}_{s,i-1}.$$
(12)

where k_i are some positive gains for i=1,...,N, agent i will approach virtual agent i as time goes to infinity. Note that when i=1, $(x_{s,i-1}, y_{s,i-1})$ will be the position vector of the target.

Consider the relationship between $(x_{s,i-1}, y_{s,i-1})$ and $(x_{s,i}^0, y_{s,i}^0)$ one has

$$x_{s,i}^{0} - x_{s,i-1} = d_{0,i}\cos(\varphi_i + \beta_{0,i}) - d_i\cos(\varphi_i + \beta_i),$$

$$y_{s,i}^{0} - y_{s,i-1} = d_{0,i}\sin(\varphi_i + \beta_{0,i}) - d_i\sin(\varphi_i + \beta_i).$$
(13)

Thus, the formation control (10) is given by

$$\begin{cases} v_{s,i} = \frac{k_i}{\cos \beta_{0,i}} (d_i \cos(\beta_i - \beta_{0,i}) - d_{0,i}) \\ + \frac{v_{s,i-1}}{\cos \beta_{0,i}} \cos(\varphi_i + \beta_{0,i} - \varphi_{i-1}), \\ w_i = \frac{k_i}{d_{0i} \cos \beta_{0i}} (d_i \sin \beta_i - d_{0,i} \sin \beta_{0,i}). \end{cases}$$
(14)

However, in many real applications, it is difficult to measure the neighbors' velocities. An assumption is made for all agents and the target:

Assumption 1: For agent i (i = 1,...,N), the velocity $(\dot{x}_{s,i},\dot{y}_{s,i})^T$ is bounded by

$$\sqrt{\dot{x}_{s,i}^2 + \dot{y}_{s,i}^2} \le v_s ,$$

for some positive number v_s . For the target, the velocity $^{(\dot{x}_0,\,\dot{y}_0)^T}$ is bounded by

$$\sqrt{\dot{x}_0^2 + \dot{y}_0^2} \le v_g$$

for some positive number v_g .

Under Assumption 1, we ignore the velocity of agent i-1 in (12), then one has

$$\dot{x}_{s,i}^{0} = -k_{i}(x_{s,i}^{0} - x_{s,i-1})$$

$$\dot{y}_{s,i}^{0} = -k_{i}(y_{s,i}^{0} - y_{s,i-1})$$
(15)

Then, simplified formation controllers are given by

$$\begin{cases} v_{s,i} = \frac{k_i}{\cos \beta_{0,i}} (d_i \cos(\beta_i - \beta_{0,i}) - d_{0,i}) \\ w_i = \frac{k_i}{d_{0i} \cos \beta_{0i}} (d_i \sin \beta_i - d_{0,i} \sin \beta_{0,i}) - \dot{\beta}_{0,i} \end{cases}$$
(16)

where the dynamics of $\dot{\beta}_{0,i}$ is given in (11).

IV. STABILITY ANALYSIS

In this section, some main results are established for the formation stability.

Theorem 1: For a group of autonomous agents with kinematics (1) and a target with disturbed dynamics (2) and (3), if formation control (14) is applied to each agent, then, for any initial values $\beta_i(0)$ and $\beta_{0,i}(0)$ ($\in [-\pi, -\pi/2) \cup (-\pi/2, \pi/2) \cup (\pi/2, \pi]$), the k-coverage formation problem is solved, that is,

$$d_i - d_{0,i} \to 0$$
, $\beta_i - \beta_{0,i} \to 0$
as $t \to \infty$.

Proof: For system (12), let $e_x^i = x_{s,i}^0 - x_{s,i-1}$, $e_y^i = y_{s,i}^0 - y_{s,i-1}$, then

$$\begin{cases} \dot{e}_{x}^{i} = \dot{x}_{s,i}^{0} - \dot{x}_{s,i-1} = -k_{i}e_{x}^{i}, \\ \dot{e}_{y}^{i} = \dot{y}_{s,i}^{0} - \dot{y}_{s,i-1} = -k_{i}e_{y}^{i}. \end{cases}$$
(17)

Then, for any positive gain k_i , one has

$$\lim_{t\to+\infty}e_x^i(t)=0, \lim_{t\to+\infty}e_y^i(t)=0.$$

From the fact:

$$\begin{split} d_{i,0} - \sqrt{e_x^i(t)^2 + e_y^i(t)^2} & \leq d_i \leq d_{i,0} + \sqrt{e_x^i(t)^2 + e_y^i(t)^2} \,, \qquad \text{one} \qquad \text{has} \\ \left| d_i - d_{0,i} \right| & \to 0 \quad \text{as} \quad t \to \infty \,. \end{split}$$

On the other hand, due to the following relationship $\tan(\beta_i + \varphi_i) = \frac{d_{0,i} \sin(\beta_{0,i} + \varphi_i) - e_y^i(t)}{d_{0,i} \cos(\beta_{0i} + \varphi_i) - e_x^i(t)},$

one has $\lim_{t\to\infty} \beta_i = \beta_{0,i}$. Thus, the conclusion follows. The proof is completed.

Theorem 2: Under Assumption 1, if formation control (16) is applied to each agent, $\beta_i(0)$ and $\beta_{0,i}(0)$ $(\in [-\pi, -\pi/2) \cup$

 $(-\pi/2,\pi/2)\cup(\pi/2,\pi])$, then, the bounded k-coverage formation problem is solved, that is,

$$\left| d_i - d_{0,i} \right| \le \varepsilon_1$$
 , $\left| \beta_i - \beta_{0,i} \right| \le \varepsilon_2$

for positive constants \mathcal{E}_1 , \mathcal{E}_2 as $t \to \infty$.

Proof. For system (15), one has

$$\begin{cases} \dot{e}_{x}^{i} = -k_{i}e_{x}^{i} - \dot{x}_{s,i-1}, \\ \dot{e}_{y}^{i} = -k_{i}e_{y}^{i} - \dot{y}_{s,i-1}. \end{cases}$$
(18)

Under Assumption 1, the velocity of agent i-1 is bounded, thus,

$$\begin{cases} |e_{x}^{i}| \leq e^{-k_{1}t} |e_{x}^{i}(0)| + \frac{\max(v_{s}, v_{g})}{k_{1}} \\ |e_{y}^{i}| \leq e^{-k_{1}t} |e_{y}^{i}(0)| + \frac{\max(v_{s}, v_{g})}{k_{1}} \end{cases}$$

Then, there exists a T, for any t>T, one has $|d_i-d_{0,i}| \leq \varepsilon_1 = \sqrt{2} \frac{max(v_s,v_g)}{k_i}, \text{ and } \left|\beta_i-\beta_{0,i}\right| \leq \varepsilon_2.$ The proof is thus completed.

V. SIMULATION RESULTS

In this section, some numerical simulations are presented to

validate the proposed target tracking algorithms (14) and (16).

Given the exogenous system

$$\begin{cases} \dot{\varsigma} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes I_2 \varsigma, \\ u = [I_2, 0] \varsigma, \end{cases}$$
(19)

The initial values of the target dynamics are given as follows:

$$p_0 = (0,0,1,1)^T$$
; $\zeta_0 = (0,0,0,-1)^T$.

The Heun Method is used to obtain the numerical computational scheme for system (8) with step h = 1/40.

Example 1: Four mobile agents with radius r = 1.2 are used to track the target and the initial distances, orientation angles, bearing angles, desired distances and bearing angles are given as follows:

$$\begin{split} &d_1(0)=1\ ,\quad d_2(0)=2\ ,\quad d_3(0)=3\ ;\quad d_4(0)=2;\ \varphi_1(0)=\pi/3\ ,\\ &\varphi_2(0)=\pi/4,\quad \varphi_3(0)=\pi/6;\quad \varphi_4(0)=\pi/3;\qquad \beta_1(0)=\pi/3,\\ &\beta_2(0)=-\pi/3,\qquad \beta_3(0)=\pi/4,\qquad \beta_4(0)=5\pi/7;\\ &d_{0,1}=2,\qquad d_{0,2}=4,\qquad d_{0,3}=4,\qquad d_{0,4}=4;\\ &\beta_{0,1}(0)=\pi,\ \beta_{0,2}(0)=-\pi/3,\ \beta_{0,3}(0)=\pi/3,\quad \beta_{0,4}(0)=2\pi/3; \end{split}$$

Formation controllers (14) and (16) with $k_1=k_2=k_3=k_4=20$, are used to make the agents form an equilateral- triangle formation with respect to the target, respectively. From Figs.3 and 4, the tracking errors of $d_i-d_{0,i}$ and $\beta_i-\beta_{0,i}$ all approach 0 with controller (14) while $d_i-d_{0,i}$ and $\beta_i-\beta_{0,i}$ are bounded with controllers (16) in Figs.5 and 6.

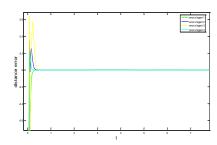


Fig.3 the distance errors evolution with controllers (14)

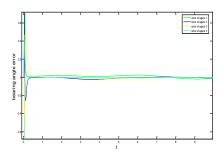


Fig.4 the bearing errors evolution with controllers (14)

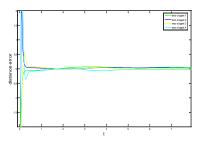


Fig.5 the distance errors evolution with controllers (16)

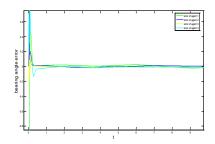


Fig.6 the bearing angle errors evolution with controllers (16)

Example 2: Controllers (16) is used to track the target under the following situations:

$$(1)^{\dot{\beta}_{0,i}} = 0$$
;

(2) $\beta_{0,i}$ is described in (11) with $\mu = 0.9$. The initial values are the same as given in Example 1.

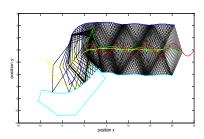


Fig.7 The formation control with the target (the red line) with $\dot{\beta}_{0,i} = 0$

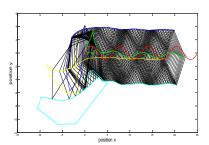


Fig.8 The formation control with the target (the red line) under Remark 1

From Figs.7 and 8, it is found that the tracking trajectories of four agents with dynamic $\beta_{0,i}$ are smoother than that with a constant $\beta_{0,i}$.

VI. CONCLUSIONS

In this paper, a target tracking problem with a group of nonholonomic agents was solved by proposing an equilateral-triangle neighbor-based formation control to improve the tracking performance. The formation stability was analyzed with a virtual agent method. Simulation results were provided to verify the proposed tracking algorithms.

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